

The time travel paradox

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Abstract

We define the time travel paradox in physical terms and prove its existence by constructing an explicit example. We argue further that in theories — such as general relativity — where the spacetime geometry is subject to nothing but differential equations and initial data no paradoxes arise.

1 Introduction

Over the last decade the (im)possibility of creating a time machine has been animatedly discussed in the literature (see [1, 2, 3] for reviews). The obvious reason is that after a specific (however unrealistic from the technical point of view) recipe for building a time machine was proposed in the noted paper [4] the problem of causality violations drastically changed its status. Until then causality in classical gravity had been perceived as something like an additional postulate. There is no observational evidence that we live in an acausal (Gödel’s, say) universe, so the problem generally had been dismissed as purely academic. The paper, however, drew attention to the fact that within general relativity causality is not given once and for all. Even if the Universe is causal at the moment it does not mean that it cannot be *made* acausal in the future (by some advanced civilization). One might not have been interested in achievements of a hypothetical civilizations, but the very fact was worrisome that one could not predict the outcome of a simple thought experiment (the manipulations with a wormhole proposed in [5]), or rather that the predicted result (appearance of a time machine) was considered by many as inappropriate.

The attempts to quickly put the jinnee back in a bottle failed. We understand now that causality can be protected neither by a shortage of ‘exotic matter’, nor by the instability of the Cauchy horizons, as it was initially suspected [6]. Indeed, exotic matter is necessary [6] for the creation of time machines (TMs) with compactly generated Cauchy horizons (CTMs), but it was shown that quantum effects can produce such matter in amounts sufficient to sustain a traversable wormhole [7] and hence a wormhole based time machine. Besides, CTMs are only a particular type of the time machines. No reasons are known

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(see [8, 3] and section 4) to consider them as something ‘better’ (more physical, or more feasible) than the TMs with non-compactly generated Cauchy horizons (NTMs) while the latter often do not require exotic matter at all. Similarly, nothing suggests that NTMs must suffer the instability. And even the CTMs while exhibiting, indeed, the instability in a few cases, in a few other do not (see [9] for a discussion). There remains, of course, the possibility that each (type of the) time machine is impossible by its own reasons, but it does not look too plausible.

Thus it seems appropriate now to turn attention to the most controversial issue related to the time machines — the time travel paradoxes. On the one hand, these paradoxes seem to be something inherent to time machines (their main attribute, perhaps), so it is reasonable to assume that if there exists a universal law prohibiting the time machines, it must have something to do with the paradoxes. And on the other hand, be the problem of paradoxes satisfactorily solved there probably would be no need to look for such a law, the (supposed) paradoxicalness of the time machines being traditionally the main objection against them.

The aim of this paper is to consider the time travel paradoxes in full detail. Two questions will be our primary concern:

1. Are the time machines associated with any paradoxes in the first place?

The answer, which once seemed obvious (the famous grandfather paradox was formulated 70 years ago and is known now in dozens of versions, see [2]), in the last decade has been getting less and less clear. The point is that an argument like ‘A time traveler can kill his grandfather before the former’s father was conceived, but then the traveler would not be born, so nobody would kill his grandfather, etc.’ while suggesting, of course, that time travel may be attended by paradoxes does not *prove* it. In particular, we have no way of modeling this situation with the necessary accuracy and so plenty of room remains for all sorts of resolutions which are abundant in science fiction (traveler can change his mind, or can kill a wrong person, etc.). To find a non-trivial paradox thus one needed a much simpler story, which could have been modeled in such detail as to take into account all relevant factors. One possibility was the Polchinski paradox [10] — a ball gets into a time machine, travels to the past and hits its younger self so that the latter misses the time machine and thus cannot later hit itself, etc. A few attempts were made (see [11, 12] for example) to construct a specific example of this paradox. For a wormhole based time machine [11] the problem still turned out to be too hard mathematically, but for the Deutsch-Politzer spacetime a paradox eventually was claimed to be found [12]. Later, however, it was shown [13] that this paradox is only *apparent*: it is not the causality violation that leads to the paradox, but some illegal global condition imposed on the physical content of the model (see the end of subsection 2.4).

Thus though it is widely believed that the existence of time machines must lead to some paradoxes in fact not a single such paradox has ever been found. Moreover, in the only model where (due to its exceptional simplicity) the ques-

tion has been fully analyzed the paradoxes were found to be lacking [13]. So, it might appear that time travel paradoxes in fact do not exist. In this article we argue that this is not the case.

We start by giving in section 2 a definition of the time travel paradox. This is necessary because a great variety of different things are called paradoxes in the literature (undeservedly, as a rule). For example, the terms ‘person’, ‘one borne by a woman’, or ‘one whose grandfather was not killed in infancy’ are essentially synonyms. So the question ‘Why a time traveler (or whoever else) did not manage to kill the traveler’s grandfather?’ is not a bit more meaningful than the question ‘Why is a ball round?’ Still, after all such ‘pseudoparadoxes’ are ruled out a situation remains for which the name ‘paradox’ is fully justified. The paradox appears as the inconsistency (due to local physical laws) of what takes place in a *causal* region with the fact that the time machine will come into existence. Thus it is not the grandfather paradox, but rather a story about a mad scientist who builds a time machine intending that after it is finished (say, on Thursday) he will commit a bizarre suicide — on Saturday he will enter the time machine, return to the Friday, and shoot his younger self dead¹.

In section 3 we build a sufficiently simple toy model (it is a flat universe populated exclusively by pointlike massless particles) and use it to prove the existence of the paradoxes by constructing a specific example.

2. How to resolve the paradox? In this paper we stay strictly within the framework of classical general relativity². The evolution of a spacetime in this theory is fundamentally non-unique. As we argue in section 4 this non-uniqueness does not let the time travel paradoxes into general relativity — whatever happens in a causal region, a spacetime always can evolve so that to avoid any paradoxes (at the sacrifice of the time machine at a pinch). The resulting spacetimes sometimes (see example 4) curiously remind one of the many-world picture.

2 The time travel paradox

In this section I specify exactly what I call the time travel paradox. The final definition will be given in subsection 2.4 after the components of the initial (rough) definition formulated in subsection 2.1 are analyzed.

2.1 Reformulation of the problem in physical terms

A great many seemingly paradoxical situations are discussed in the literature (one even can encounter the ‘sexual paradoxes’ separated into a special category [2]). It is important for our purposes, however, that they all amount to the (assumed) fact that

¹That is (in terms of killing grandfathers) the interesting question is ‘Why is it that a child who plans, first, to build his time machine (at the age of 20), then to marry (at 30), and then to force his offspring to kill him when he is 25, will always fail?’

²Correspondingly, we deal neither with the (quantum) ‘many-world’ theory [14], nor with the (metanomological [15]?) ‘principle of self-consistency’ [10].

due to the presence of a time machine a system (an elastic ball, an armed time traveler, etc.) *has a state* (the ball moves with a given speed in a given direction, the traveler meets his younger self, etc.) *incompatible with the laws governing the evolution of the system* (the ball appropriately deviates when struck, the person kills whoever he sees, etc.)

This formulation still needs a lot of refinements of course, but at the moment the following should be emphasized. By replacing the ‘circling’ part of the story (‘the ball hits its former self, so it does not enter the time machine, so it will not be hit, so it enters, etc.’) with the plain statement that the laws of motion of the ball *are incompatible* with its initial state we have not lost anything paradoxical. Indeed, that mind-boggling circle is nothing more than a proof (by contradiction) of the incompatibility: ‘Suppose the ball evolves from that initial state according to these laws, then it would have to hit its former self, ... q. e. d.’

All known (to me) time travel paradoxes fit in the above formulation. Whatever interesting there is in science fiction beyond the above statement (a traveler returning to a world different from what he remembers, or the universe disappearing in order not to allow of a paradox, etc.), it all relates to ‘why is it a paradox?’, or ‘how to cope with paradoxes?’, but not to ‘what kind of situations should be called the time travel paradoxes?’ In particular, I shall not consider the so-called ‘bootstrap paradoxes’ [1] as a separate class. Such paradoxes can be represented, for example, by a story about an engineer, who departs to the future, reads the patent disclosures on some device, returns to ‘his’ time and does patent it [16]. Where did the idea of this invention come from? In another version [17] the protagonist receives a note (with a very helpful clue) from his older self, keeps this note in his wallet and when (his) time comes hands the note to the addressee — to his younger self. Who wrote the note? Generally, this class comprises the situations in which the initial data are compatible with the equations of motion, but the solutions look counterintuitive by whatever reason (often because a ‘lion’ (see below) is necessary to achieve the compatibility). Confronting a bootstrap paradox we have a choice: either we find a specific law which is violated (converting thus the bootstrap paradox into an ordinary one), or we must admit that the situation — however strange it might be — is not a paradox at all.

2.2 The requirements on the laws of motion

An important part of the definition in question are the words ‘due to’. Indeed, it seems unreasonable to call the incompatibility of an initial state with laws of motion ‘a time travel paradox’ unless we have at least *some* reason to believe that this incompatibility takes place *due to* the presence of a time machine and not just *in* its presence. The natural criterion for distinguishing the two situations would be whether or not arbitrary initial data are compatible with these laws in a *causal* world.

Thus, in defining the time travel paradox it makes sense to restrict ourselves to the laws that are not ‘paradoxical’ in the causal worlds. Before formulating the relevant condition, however, we have to solve a preliminary problem. The point is that generally one cannot compare the laws of motion in two different spacetimes (a causal and an acausal in our case).

Example 1. Take the Minkowski plane and make the cuts along the two spacelike segments $\{t = \pm 1, -1 < x < 1\}$. Remove also the ‘corner’ points ($t = \pm 1, x = \pm 1$). Now *preserving their orientation* glue the banks of the cuts — the upper bank of the lower cut to the lower bank of the upper cut and vice versa. This surgery results in a spacetime (see figure 1a) called the 2-dimensional Deutsch-Politzer (DP) time machine. The corner points cannot be glued back into the spacetime and thus the DP space has singularities.

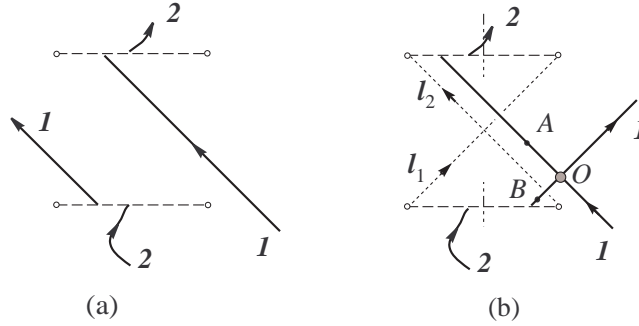


Figure 1: The two-dimensional Deutsch-Politzer space (a) and its twisted version (b). An extension of any continuous curve approaching a (former) cut must start from the corresponding bank of the other (former) cut, which looks as discontinuity in the picture.

Remark 1. In the 4-dimensional case the procedure is the same, but the cuts are made along the cubes $Q_{1(2)} \equiv \{t = \pm 1, -1 < x, y, z < 1\}$ instead of the segments. Correspondingly, the singularity in this case appears as a result of removing the 2-dimensional boundaries of the cubes (not just four points).

Remark 2. The DP space is sometimes regarded as something contrived and unphysical (an attitude which is hard to substantiate within the regular general relativity, cf. section 4). In this connection note that the surgery by which we obtained the DP space is just a convenient way to describe its structure. One can as well define the Minkowski space as result of an appropriate surgery applied to the DP space.

Now consider the two-dimensional Deutsch-Politzer spacetime populated by particles of two kinds. Let us call them s - and d -particles and depict their world lines with single and double lines respectively. We assign a vector (‘momentum’) to each particle — null to an s -particle and timelike to a d -particle — and require the world line of any particle to be a (segment of) straight line parallel to its

momentum (physically speaking, we describe a world with two types of particles — massive and massless — and with pointwise interaction). The laws of motion will be the following. The world lines can terminate only in vertices. Outside a small region \mathcal{R} around the origin of the coordinates (the shadowed region in figure 2b) the particles do not interact (an intersection of world lines does not make a vertex). Inside \mathcal{R} there are vertices of two types (see figure 2a): (1) if two s -particles with the momenta \mathbf{p}_1 and \mathbf{p}_2 and a d -particle with the momentum $\mathbf{p}_1 + \mathbf{p}_2$ meet in some point, all particles colliding in this point annihilate; (2) if among the colliding particles there are two s -particles, but the vertex is not of type 1 (no d -particle with the ‘correct’ momentum) then the outcome is a single d -particle with the momentum $\mathbf{p}_1 + \mathbf{p}_2$.

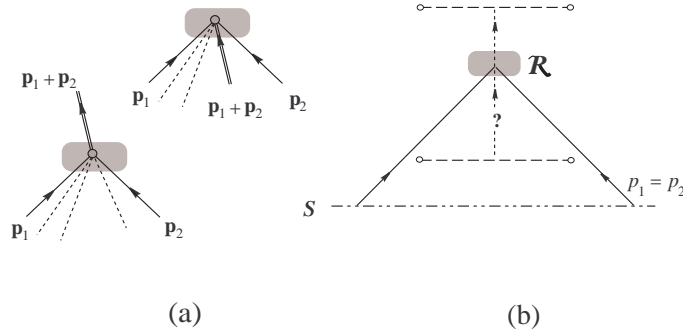


Figure 2: The local laws (a) are inconsistent with the initial data fixed at S .

It is easy to construct a paradox in the described world. For example, a system of two s -particles with the initial data shown in figure 2b obviously cannot obey the laws formulated above.

We cannot know, however, what would happen with these particles in a causal world (the laws of motion cannot be transferred to any other spacetime, since, for example, we cannot distinguish the analog of \mathcal{R} there) and so there are no reasons to attribute this ‘paradox’ to the time machine rather than to intrinsic pathologies of the postulated laws of motion.

To avoid such situations it suffices to subject the laws of motion to the following condition³.

C1 (Locality). The laws of motion inside any region U do not depend on anything outside of U .

To put it more specifically, (C1) requires that in determining whether or not a system confined to U obeys a law the answer should not depend on whether U can be extended to a larger spacetime U' and what are the physical conditions

³Note that in general relativity a much more restrictive condition called *local causality* [18] is normally accepted.

in $U' - U$. Events in $U' - U$ can influence those in U only via the boundary conditions. The laws satisfying (C1) we shall call *local*.

Remark 3. Condition (C1) rules out, in particular, theories in which non-locality originates from incompleteness. One might say, for example, that the region \mathcal{R} in example 1 is just a way of description of some field (governed by local laws) the interaction between particles being dependent on the value of this field. Then to make the model complete one would have to explicitly include the equations of the field in it, which perhaps would remove the paradox.

Remark 4. It is not only the presence of \mathcal{R} that contradicts (C1) in the above example. Locality implies among other things that if some vertex exists in a model than there must also exist all vertices obtained from it by (in the flat case) Lorentz transformations and this does not hold in the example (see figure 2a). Note that if we make the physics of our model local by extending \mathcal{R} to the whole spacetime and by adding the missing vertices (in particular the

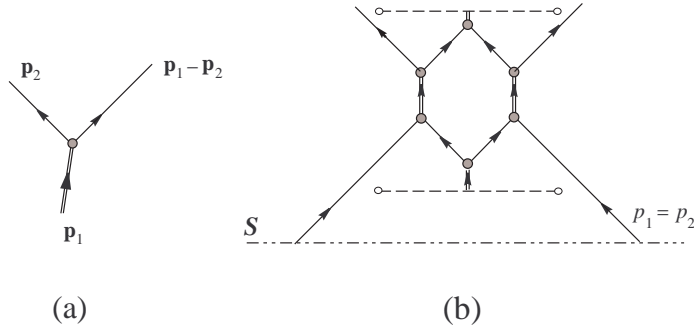


Figure 3: When the physics of example 1 is modified so that it becomes local, the paradox disappears.

one shown in figure 3a) the paradox disappears (see figure 3b).

When the condition (C1) holds the comparison of evolutions in a causal and an acausal spacetimes becomes meaningful (we always can cover a spacetime by neighborhoods $\{U_\alpha\}$, where each U_α is causal) and by the reasons discussed above we impose yet another condition:

C2. In any *causal* spacetime⁴ the laws of motion must be compatible with any initial data.

The following example shows that this condition is actually more restrictive than it might seem.

Example 2. Consider the massless field in the Misner space (which is the Minkowski half-plane $ds^2 = -dudw$, $w < 0$ with the points identified by the rule $(u_0, w_0) \mapsto (\mu u_0, w_0/\mu)$, see figure 4). It is easy to show (cf. [19]) that

⁴In particular, in any U_α , which shows the non-local nature of the time travel paradoxes.

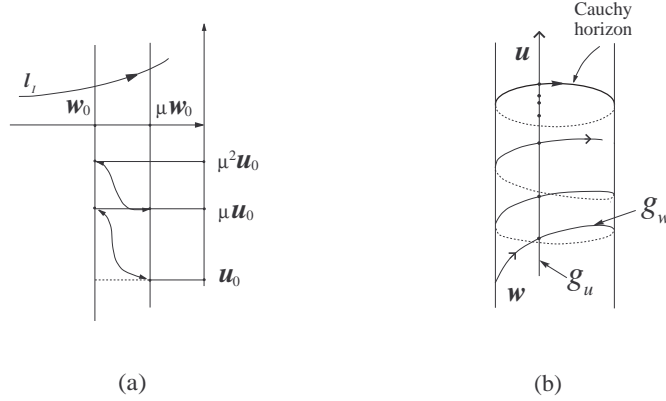


Figure 4: The Minkowski half-plane (a) after its points are identified as shown with arrows becomes the Misner space (b) coordinatized with $u \in \mathbb{R}$, $w \in (\mu, 1]$. Its lower part ($u < 0$) is globally hyperbolic, and the upper part is acausal.

the only smooth solution in this space of its equation of motion $\square\phi = 0$ is $\phi = \text{const}$ [indeed, the null geodesics g_u and g_w intersect infinitely many times (see figure 4b) before the former reaches the Cauchy horizon, and each time $\partial_u\phi$ increases by the same factor μ^{-1}], which obviously is incompatible with generic initial data. However, one can hardly use this situation as a model for a time travel paradox, since this field may not possess an evolution (for given initial data) even in causal (though not globally hyperbolic of course) spacetimes as well. An example of such a spacetime is the Misner space with a ray ($w = w_0, u \geq 0$) removed from the acausal part.

2.3 The lack of information in causal regions

In the next subsection we shall return to the discussion of the definition, but now let us dwell on a fundamental, though sometimes overlooked property of time machines. The systems in consideration are classical. So, intuition (based on what takes place in globally hyperbolic spacetimes) might suggest that by fixing their state ‘at some moment’ (that is on a spacelike hypersurface S) we uniquely fix their evolution. But in non-globally hyperbolic spacetimes *this is not the case*. In any acausal region there always are inextendible nonspacelike curves which do not originate from a causal region. Those are the closed curves (e. g. l_L in figure 5) and the curves intruding the spacetime ‘from nowhere’ (e. g. l_n in figure 1b and l_I in figures. 4a, 5). Along such curves some ‘extra unpredictable’ (to an observer getting into the time machine from the causal region) information can enter the spacetime (cf. [6]). These curves may be the world lines of some particles that neither existed prior to the time machine, nor originated from only the ‘pre-existing’ particles. The unusual origin of the ‘un-

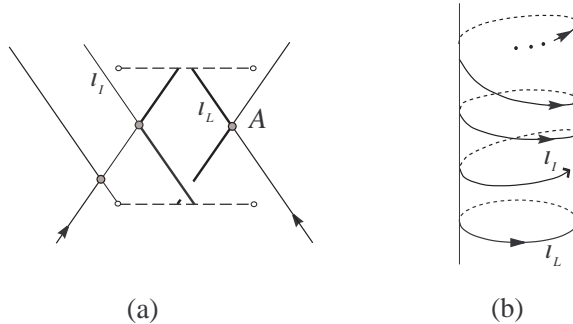


Figure 5: Different types of lions in the DP (a) and Misner (b) spaces. l_L loop in the acausal regions and l_I intrude the spacetimes from a singularity (a), or from infinity (b). (The latter is the same object as that in figure 4a.)

expected' particles does not make them any less physical than the conventional ones (which, after all, also could appear only from either infinity, or a singularity) and so in our considerations we must take the particles of both kinds equally serious. The more so, as the line between 'looping' particles⁵ l_L and 'normal' ones is not always drawn absolutely clear. For example, l_L in figure 5a can be interpreted as a normal particle emitted in A and later (by its clock) absorbed there.

The fact that entering a time machine one can meet there an object whose existence it was impossible to predict is of great importance not only for resolutions of particular paradoxes (an unexpected hungry lion behind the door of a time machine could effectively reconcile the traveler's freedom of will and his grandfather's safety), but for the very concept of the paradox. Indeed, most if not all of paradoxes proposed in the literature are based on some (implicit, as a rule) assumptions about presence of looping or intruding objects — from now on I shall use abbreviation *lions* for them — and their properties. For example, the puzzling piece of paper figuring in [17] (see subsection 2.1) is a typical lion. Another example (cf. [12]) is the paradox resulting from the assumption that *no* lions at all will appear in the acausal region of the DP space when a pair of elastic balls is prepared in the initial state shown in figure 5a by arrows (see [13] for more details).

2.4 The definition of the paradox

A possible way to obviate the problems with uniqueness of evolution would be to fix the 'initial data' (they are not initial data in the ordinary sense as we argue below) for all relevant particles *including* lions. In the case of the DP space for

⁵Aka 'self-sufficient loops' [17], 'jinn' [20], 'self-existing objects' [15].

example, it could be done (see figure 6a) by choosing the surface $t = 0$ to be the ‘initial surface’. This must be done with caution, however. Such a surface is not *achronal*, i. e. some points in it can be connected by timelike curves (that is why data fixed there cannot be rightfully called initial). To be consistent with the evolution laws these data must satisfy some constraints even in theories free from any paradoxes.

Example 3. Consider the DP space populated by stable non-interacting particles. The ‘initial data’ shown in figure 6a (a single particle moving at $t = 0$ to the right) are inconsistent with any possible evolution. To obtain a ‘paradox’ of precisely the same nature in a globally hyperbolic spacetime take the surface $t = \varphi/2$ as the initial surface in the cylinder $t \in \mathbb{R}, \varphi \in (0, 1]$ with the metric $ds^2 = d\varphi^2 - dt^2$, (see figure 6b).

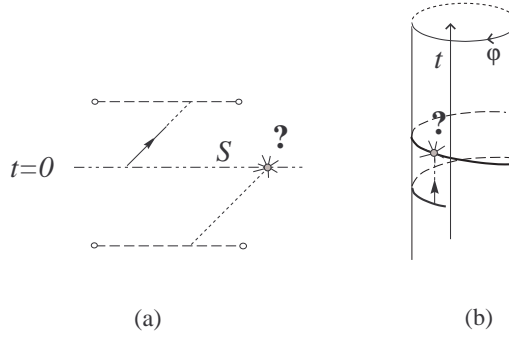


Figure 6: Trivial paradoxes in an acausal (a) and in a globally hyperbolic (b) spacetimes. No evolution of a freely moving particle is consistent with the initial data depicted by the arrows.

To exclude such trivial ‘paradoxes’ we (at the sacrifice of uniqueness of evolution) shall fix the initial data in the proper way — at achronal surfaces (and thus outside of the acausal region). Correspondingly, we adopt the following final definition of the paradox:

Definition. The paradox is the inconsistency of some laws of motion satisfying conditions (C1,C2) with initial data fixed in a causal region.

It should be stressed that in doing so we do not miss anything really paradoxical. Indeed, suppose we find that a time traveler inside a time machine is prohibited from doing something (say, from killing a younger person he meets there) that he would be allowed to do if exactly the same situation (the traveler in the same mood embedded in exactly the same local environment) took place in a causal region. Not observing any mechanism that could enforce the prohibition one might want to call the situation paradoxical.

Consider, however, an adventurer who wants to build such a time machine and to try to violate the prohibition — to kill his younger self (just in order to find out what can prevent him from doing this). The adventurer, when he is in the causal region yet, is free to make any preparations he wants. He can choose whatever good weapon, he can adopt any strategy for his behavior inside the time machine, etc. — lost labor. There always will remain an evolution (i. e. a history not containing the paradoxical suicide) consistent with all these preparations (or, otherwise there would be a paradox in the sense of the above definition). What will be the immediate cause of his failure (whether he will be eaten by a lion, or just miss the target) depends on the details of the situation and is immaterial (as long as no new local physics is involved, which is supposedly the case). What counts is that he is allowed to make his attempt and so his life does not cost him his freedom of will⁶. Thus it would be unjustified to call such a situation a paradox.

Remark 5. The ‘trivial paradoxes’ (see example 3) that we ruled out by adopting our definition constitute the vast majority of what is traditionally considered as paradoxes (these are the grandfather paradox and its numerous modifications). Still our understanding of what is the time travel paradox is not new: ‘... if there are closed timelike lines to the future of a given spacelike hypersurface, the set of possible initial data for classical matter on that hypersurface can be heavily constrained compared with what it would be if the same hypersurface with the same local interactions were embedded in a chronology-respecting spacetime’ [14].

3 A paradox

3.1 The geometry

The simplest model of the time machine — the DP space — unluckily for our purpose does not harbor paradoxes (at least when the physics is simple enough) [13]. So for a paradox we have to choose a spacetime with a slightly more complex geometry.

Perform the same manipulations as in constructing the DP space (see example 1), but this time glue the upper bank of the lower cut to its counterpart only after it is *reflected* with respect to the t -axis. In other words a point with $x = x_0$ of this bank is now glued to the point with $x = -x_0$ of the other bank (not to the point with $x = x_0$ as it was in the DP case). The resulting spacetime — let us call it *the twisted Deutsch-Politzer (TDP) space* — is non-orientable⁷ though still time-orientable, of course. Note that in the TDP space a null geodesic (see curve 1 in Fig 1b) entering the time machine always has a self-intersection.

⁶This reasoning does not work if the adventurer is a lion, but I do not think that a lion’s freedom of will should be anybody’s concern.

⁷The TDP space is in fact the DP space with a cylinder replaced by a Möbius strip.

3.2 The physics

The world in our model is populated by massless pointlike particles (that is particles moving along null geodesics that terminate only in vertices). Two parameters are assigned to each particle — the ‘color’ (c) and the ‘flavor’ (f) with the possible values

$$c = b, g, r \quad f = \pm 1$$

We write c^f (or g^{-1} , b^1 , etc.) for a particle with the color c (respectively, g , b) and the flavor f (respectively, 1, -1). The particles do not interact (an intersection of their world lines does not make a vertex), with a single exception — when two particles of the same kind (i. e. of the same color and flavor) meet, they change their flavor (see figure 7).

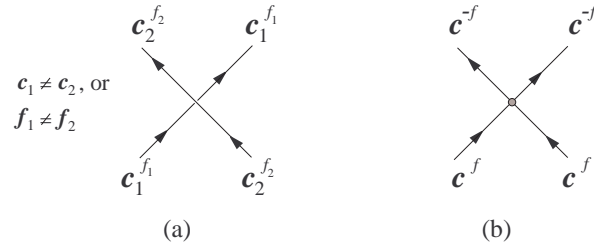


Figure 7: The laws of motion guarantee that in pair collisions the left outgoing particle differs from the left incoming one.

Remark 6. If required, one can adopt another (though equivalent in the case at hand) point of view and speak not about penetrable particles, but about particles that bounce from each other with their parameters changed according to figure 7.

3.3 The paradox

Let us consider the system with the initial data posed as shown in figure 8: at some moment (the surface S) preceding the appearance of the time machine there are three particles — of three different colors — moving so that they must get into the time machine.

These initial data do not allow any evolution, indeed. To see this consider the would-be trajectories of the particles after their collision with the lion l_4 . There remain only two lions (l_1 and l_2) that can be met on their way. So, there will be at least one particle — let it be the red one ($c = r$) for definiteness — which will not collide any more with a lion of the same color:

$$c(l_1) \neq r \neq c(l_2)$$

Its world line (if it existed) would be the geodesic 1 in figure 1b. It is easy to see that no flavor can be assigned to the particle on the segment OA . Indeed, on

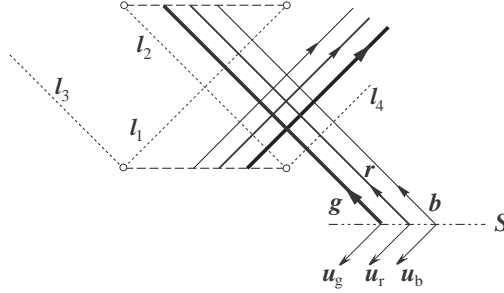


Figure 8: All possible trajectories are shown kinematically compatible with the data fixed at S .

the (open) segment ($OABO$) of its world line it does not experience collisions with other red particles and hence its flavor cannot change. Hence the flavor – along with the color – on the segment BO must be the same as on AO , which is prohibited by the local physics (see figure 7b).

Remark 7. Insofar as we discuss *penetrable* particles it is not always possible to assign a particular color to each world line. Two or three particles with the different colors may have the same world line (it can be, for example, any of the geodesics l_i in figure 8) forming thus a ‘composite’ particle. Such a particle, however, will be sterile. According to the laws specified in subsection 3.2 interaction occurs only when *two* particles (with the same colors) meet and hence a composite particle passes through any other particle (composite, or not) causing no changes.

3.4 A four-dimensional version

The paradox constructed in the previous subsection satisfies all our requirements. However, it has two (slightly) objectionable features. First, the spacetime is non-orientable. Second, the transformation $x \leftrightarrow t$ though not being an isometry still strongly resembles it. So, appealing to condition (C1) one might argue that the existence of the vertex $c^f + c^f \rightarrow c^{-f} + c^{-f}$ (see figure 7b) should imply the existence of the vertex $c^f + c^{-f} \rightarrow c^f + c^{-f}$, which would destroy the paradox. We shall show now that in the four-dimensional case both these ‘flaws’ can be eliminated.

To build the spacetime that we shall use, repeat the construction of the four-dimensional DP space as is described in the beginning of section 2.1, but before gluing the cubes Q_1 and Q_2 rotate one of them by π in the (x, y) plane. The resulting spacetime M is orientable and the transformation $x \leftrightarrow t$ does not even resemble an isometry any longer. At the same time the two-dimensional section ($y = z = 0$) of M is exactly the TDP space.

We cannot construct a paradox this time by simply using the same initial

conditions and physics as in the two-dimensional case. The singularities now are formed by the (former) boundaries of the cubes. So, there may be infinitely many lions colliding with the initial particles, in contrast to the 2D case where there were only 3 such lions. To handle this problem we shall slightly modify the local physics. Namely, in addition to color and flavor we assign to each particle a null vector \mathbf{u} which parallel propagates along the world line of the particle and does not change in collisions. We require that particles 1 and 2 should not interact unless

$$\mathbf{u}_1 \parallel \mathbf{v}_2, \quad \mathbf{u}_2 \parallel \mathbf{v}_1, \quad (1)$$

where \mathbf{v}_i is the four-velocity of the i th particle, and when (1) holds they should interact exactly as in the two-dimensional case.

The so chosen laws guarantee that a particle with \mathbf{u} lying in the (t, x) plane can interact only with the lions whose world lines also lie in this plane. Thus restricting consideration to this plane and choosing \mathbf{u} for the three initial particles as is shown in figure 8 we reduce the problem to the two dimensional one and obtain a paradox.

4 The paradox as an argument against time machines

In the previous section we proved that the problem (the time travel paradox) really exists. Now let us discuss how to solve it.

Three things — the set of local physical laws, the initial data, and the causal structure of the spacetime — individually (supposedly) allowed, turned out to be in conflict. Correspondingly, (at least) three ways out are seen.

The system in our toy paradox was governed by exceedingly simple laws. One might conjecture that the paradox is just a result of this simplicity while the real (much more rich) physics is free from paradoxes and any initial data are admissible irrespective of the causal structure. It is impossible to *refute* (or to prove, for that matter) this conjecture, but, on the other hand, it is not obvious why a more complex and detailed theory must lead to fewer rather than to more paradoxes.

Also one can just *ignore* the paradoxes. After all only *some* (but not all imaginable) initial conditions are realized in the Universe. So, one can argue that the question ‘Why this or that particular situation is forbidden?’ is senseless, while the ‘correct’ question is ‘Whether this situation is realized in the (unique) existing Universe?’ All complications arising then with the notion of free will is a problem rather philosophical than physical. In my opinion, however, this approach is somewhat too universal to be interesting.

Finally, the third possibility is to question the feasibility of the spacetime geometry involved in the paradox. Instead of asking whether *time travel* is associated with paradoxes one can ask whether *general relativity* is associated with them. The difference between the two questions is that in answering the

latter we must consider the geometry of the universe not as a given background, but rather as a part of the system, on the same terms as the would-be time traveler. Correspondingly a paradox now must be defined as an inconsistency of some initial data fixed (in a causal region) for a material system *and* a spacetime with their laws of evolution. By the ‘laws of evolution of the spacetime’ we understand (as we restricted ourselves to classical gravity) the postulate according to which the Universe is described by a maximal (that is inextendible) time-orientable spacetime, the latter being defined as a smooth four-dimensional Hausdorff manifold with the Lorentzian metric obeying the Einstein (or other similar) equations.

The two definitions of a paradox are equivalent only as long as we neglect any influence of matter on the geometry and also believe in uniqueness of evolution of the spacetime.

That the evolution of a spacetime is not fixed uniquely by initial data and that due to this fact the existence of a time travel paradox does not necessarily entail anything paradoxical from the relativistic point of view can be immediately exemplified by the paradox constructed in the previous section. Indeed, the initial data (including the geometrical part) fixed at S are, in fact, quite *compatible* with the laws of motion (though not with all possible evolutions agreeing with these laws). In particular, the spacetime may evolve just in the Minkowski space. The three particles will fly unimpeded in this space and no paradoxes will arise.

It may appear that so can be resolved only paradoxes involving some special types of the time machine (e. g. the time machines with the non-compactly generated Cauchy horizons⁸). Indeed, the whole concept of the time machine as something made by human beings (as opposed to something appearing ‘spontaneously’ like the Deutsch-Politzer time machine) is based on the assumption [5] that one can *force* a spacetime to evolve into a time machine (cf. ‘the potency condition’ of [3]), or to put it otherwise that there exist such initial conditions that a spacetime evolving from them would inevitably produce a time machine. However, this assumption is, strictly speaking, utterly groundless within the limits of pure (i. e. without any additions to the above postulate) general relativity. In this theory *no spacetime at all* evolves uniquely. Whatever extension $M' \supsetneq M$ one takes to be a possible evolution of the initial spacetime M , there always exists another spacetime M'' — infinitely many such spacetimes, in fact — which also presents a possible evolution of M . For example, one can remove a 2-sphere Σ from $M' - M$ and take the universal covering of the resulting space $M' - M - \Sigma$ as M'' [21, 8]. Note, in particular, that locally M' and M'' are isometric and thus neither of them is preferable from the point of view of the Einstein equations⁹.

⁸This illusion is probably the reason why such time machines are sometimes regarded as ‘less physical’ than CTMs [6].

⁹It is often said that the non-uniqueness of evolution is due to the fact that the Einstein equations do not fix the *topology* of a spacetime. That is the truth, but not the whole truth. Pick another sphere Σ' and consider the universal covering M''' of $M' - M - \Sigma'$. M''' has the same topology as M'' , but (in the general case) a different geometry.

To overcome such a disastrous lack of predictability one usually restricts one's consideration to some particular class of spacetimes, that is in effect one introduces a new (additional to that formulated above) postulate in the theory. Not infrequently it is something like 'a spacetime must remain globally hyperbolic as long as possible' (among the other possibilities is the requirement that a spacetime should be hole-free). Though incorporation of this postulate (after appropriate refining) could give rise to an interesting theory we shall not consider it in the present article because, anyway, such an 'improved relativity' fails at the Cauchy horizons that bound (as in the case of the wormhole-based time machine, or the Misner space) *maximal* Cauchy developments. A spacetime has infinitely many extensions beyond such a horizon and *none* of them is globally hyperbolic.

As there are no grounds in the theory to prefer a particular class of extensions the only remaining way to construct a paradox would be to find such initial data in the causal region that *all* possible evolutions would lead to formation of a time machine in the future. This, however is impossible. The following theorem can be proved [22]: *any* spacetime M has a maximal extension containing no closed causal curves except perhaps those lying in the past of M .

Example 4. Let M be the causal part of the Misner space (see figure 4b). There are two well-known 'natural' ways [18] to extend it to the whole Misner space (i. e. to a cylinder with closed timelike curves in it). But there are also infinitely many ways to extend it to a maximal *causal* spacetime. For example (cf. [3]), let M' be the Misner space with the ray ($w = w_0, u \geq 0$) cut out from the acausal part. Take a sequence $\{M'_n\}$, $n = \dots, -1, 0, 1 \dots$ of copies of M' . For each n glue the right bank of the cut in M'_n to the left bank of the cut in M'_{n+1} . The resulting spacetime is a causal maximal extension of M .

If desired (see [23]) the abovementioned theorem could serve as basis for postulating causality. What is more pertinent to the present paper, it proves that *there are no time travel paradoxes in general relativity*: for any system governed by laws satisfying (C1,C2) any initial data are allowed in a causal region of the universe.

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